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# Transverse self-focusing and filamentation of a laser beam in a collisional magnetoplasma

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**Abstract** : Anisotropic self-focusing of a Gaussian laser beam in a collisional magnetoplasma is examined analytically. The laser propagates transverse to the static magnetic field. The nonlinearity arises through the Ohmic heating of electrons and subsequent redistribution of plasma along the magnetic field. The magnetic field enhances the rate of self-focusing to a large extent. In the case of a plane-uniform laser beam, it is seen that it is unstable for small scale fluctuations, and the growth rate of instability increases rapidly with magnetic field.

**Keywords** : Self- focusing, Gaussian-laser beam, ohmic heating, anisotropy, filamentation.

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## 1. Introduction

The self-focusing of laser beam in nonlinear medium has been a subject of many theoretical, numerical as well as experimental investigations [1–5]. Sodha and Tripathi [1,2] have studied the self-focusing of Gaussian electromagnetic beams in a collisisonless magnetoplasma when the direction of propagation is aligned along the static magnetic field. The nonlinearity in the dielectric tensor of the palsma arises through the ponderomotive force of electrons and subsequent redistribution of plasma along the wavefront. However, the process of redistribution of plasma is slowed down drastically by the magnetic field, and hence the process of self-focusing may not be realized on short time scales. Sharma [4] has studied the problem of self-focusing of a Gaussian laser beam in a collisionless magnetoplasma when the direction of wave propagation is at right angles to the magnetic field.

In this paper, we have studied the self-focusing of a laser beam in a collisional magnetoplasma. It is relevant to self-focusing of radio waves in the ionosphere and low temperature laboratory plasmas. On account of nonuniform intensity distribution of the main beam, the carriers get redistributed leading to a nonlinear dielectric tensor of the magnetoplasma. Nonuniform heating of electrons and their consequent redistribution is considered to be dominant source of nonlinearity on the long time scale ($t_o >> t_e$) in a collisional magnetoplasma [2]. When the beam is propagating transverse to the direction of the static magnetic field, two modes of propagation *viz.* extraordinary and ordinary mode exist. The analysis in this paper has also been extended to study the filamentation instability of a plane uniform laser beam when the perturbation is polarized (i) along and (ii) transverse to the magnetic field. It is seen that the beam breaks into small-scale filaments in the direction of magnetic field.

## 2. Nonlinear dielectric tensor

Consider the propagation of a nonuniform (Gaussian) electromagnetic wave of angular frequency $\omega_o$ in a homogeneous magneto plasma along the $x$ axis

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$$E = A\exp[i(\omega_0 t - k_0 x)] \quad (1)$$

The static magnetic field is aligned along the $z$ axis. For the extraordinary mode [1]

$$A = \hat{x}A_x + \hat{y}A_y, A_y = -(\in_{xx} / \in_{xy} A_x), \quad (2)$$

$$k_o = \frac{\omega_0}{C}\left(\frac{2\varepsilon_{o+}\varepsilon_{o-}}{\varepsilon_{o+} + \varepsilon_{o-}}\right)^{1/2}$$

where

$$\in_{o\pm} = 1 - \frac{\omega_p^2}{\omega_0(\omega_0 \mp \omega_c)}$$

$\omega_c$ and $\omega_p$ being the electron cyclotron and equilibrium plasma frequencies, respectively. The intensity distribution of the beam at $x = 0$ may be taken to be Gaussian. In the presence of electric vector of the beam, the electrons acquire an oscillatory velocity as [1]

$$V_x + iV_y = \frac{-e[E_x + iE_y]}{m[v + i(\omega_0 - \omega_c)]},$$

$$V_x - iV_y = \frac{-e[E_x - iE_y]}{m[v + i(\omega_0 + \omega_c)]}, \quad (3)$$

where $-e$, $m$ and $v$ are the electronic charge, mass and collisional frequency. The current density can now be expressed as [1]

$$J_x + iJ_y = \sigma_+(E_x + iE_y), \quad (4)$$

$$J_x - iJ_y = \sigma_-(E_x - iE_y),$$

and

$$J_z = \sigma_{zz}E_z,$$

where

$$\sigma_+^- = \frac{ne^2}{m[v + i(\omega_0 - \omega_c)]}, \quad (5)$$

$$\sigma_- = \frac{ne^2}{m[v + i(\omega_0 - \omega_c)]},$$

$$\sigma_{zz} = \frac{ne^2}{m(v + i\omega_0)},$$

where $n$ is the local concentration of electrons. Effective displacement vector [1] may be expressed as

$$D_{eff} = E + \frac{4\pi}{i\omega}J \quad (6)$$

or

$$(D_{eff})_i = \Sigma_j \in_{ij} E_j,$$

where $i$, $j$ = $x$, $y$, $z$ and $\in$ is the effective complex dielectric tensor having the following components.

$$_x = \in_{yy} = 1 - \frac{2\pi i}{\omega_0}(\sigma_+ + \sigma_-),$$

$$-\in_{yx} = \in_{xy} = \frac{2\pi}{\omega_0}(\sigma_+ - \sigma_-),$$

$$\in_{zz} = 1 - \frac{4\pi ne^2}{m\omega_0(\omega_0 - iv)} \quad (7)$$

and

$$\in_{xz} = \in_{zx} = \in_{yz} = \in_{zy} = 0.$$

The local electronic concentration $n$ is a function of the electric field and hence the dielectric tensor is nonlinear.

Considering the heating of electrons due to extraordinary mode ($E_z = 0$, $E_x \neq 0$ and $E_y = 0$) in a magnetoplasma, the time-independent part of power absorbed per electron from the wave is

$$-\frac{e}{2}R_e(v.E^*) = \frac{e^2}{4m}\left(\frac{A_1A_1^*}{(\omega_0 - \omega_c)^2 + v^2}, \frac{A_2A_2^*}{(\omega_0 - \omega_c)^2 + v^2}\right) \quad (8)$$

In the steady state, this power has to be dissipated in collisions with neutral particles. Then the time-independent part of electron temperature [1] comes out to be

$$\frac{T_e - T_0}{T_0} = \frac{a\omega_0^2}{2}\left(\frac{A_1A_1^*}{(\omega_0 - \omega_c)^2 + v^2}, \frac{A_2A_2^*}{(\omega_0 - \omega_c)^2 + v^2}\right) \quad (9)$$

where

$$E_x = \frac{1}{2}(A_1 + A_2), \quad E_y = \frac{1}{2i}(A_1 - A_2),$$

$$a = \frac{e^2M}{6m^2\omega_0^2K_BT_0}$$

$K_B$ is the Boltzmann's constant, $M$ is mass of heavy particles and $T_0$ is the equilibrium temperature of electrons. A corresponding variation in the electronic concentration may be expressed as

$$n = \frac{2n_0T_0}{T_e - T_0}, \quad (10)$$

where $T_e$ and $n_0$ are the electron temperature and equilibrium concentration. Electron density may be expressed as

$$n = n_0 - \Delta n, \tag{11}$$

where

$$\Delta n = \frac{a\omega_0^2 n_0}{4}\left(\frac{2\left(\omega_0^2+\omega_c^2\right)\left(E_x E_x^* + E_y E_y^*\right)}{\left(\omega_0^2-\omega_c^2\right)^2} + \frac{4i\omega\omega_c}{\left(\omega_0^2-\omega_c^2\right)^2}\left(E_y E_x^* - E_x E_y^*\right)\right)$$

$$\frac{T_e - T_0}{T_0} < 1 \text{ and } \frac{\nu}{|\omega_0-\omega_c|} < 1.$$

Using eq. (11), the components of nonlinear dielectric tensor of the plasma may be written as [2]

$$\in_{xx} \mp \in_{xy} = \in_{\pm} = \in_{0\pm} + \Phi_{\pm}, \tag{12}$$

and

$$\Phi_{\pm} = \frac{\omega_p^2}{\omega_0\left(\omega_0 \mp \omega_c\right)}\frac{\Delta n}{n_0}$$

### 3. Self-focusing of two-dimensional beam

The wave equation governing the propagation of electromagnetic waves in a plasma may be written as [1]

$$\Delta^2 E - \Delta(\Delta . E) + \left|\frac{\omega_0^2}{c^2}\right| \in . E = 0. \tag{13}$$

For the two-dimensional beam in the case of extraordinary mode ($E_z = 0$, $E_x \neq 0$, $E_y \neq 0$ and $\partial/\partial_y = 0$) the wave eq. (13) reduces to

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2} + \left(k_0^2 + \frac{\omega_0^2}{c^2}\in_2 E_y E_x^*\right) = 0, \tag{14}$$

where

$$k_0 = \omega_0 \in_{\text{eff}}^{1/2} / c$$

$$\in_{\text{eff}} = \frac{2\in_{0+}\in_{0-}}{\in_{0+} + \in_{0-}}$$

$$= 1 - \left|\frac{\omega_p^2}{\omega_0^2}\frac{\omega_0^2-\omega_p^2}{\omega_0^2-\omega_u^2}\right|$$

$$\omega_u^2 = \omega_p^2 + \omega_c^2,$$

$$\omega_p^2 = 4\pi n_0 e^2 / m,$$

$$\frac{\partial}{\partial E_y E_y^*}\left(\frac{2\in_-\in_+}{\in_+ + \in_-}\right)E_y E_y^* = 0,$$

$$= \frac{\alpha\omega_p^2\left(\omega_0^2-\omega_p^2\right)\left(\omega_0^2+\omega_c^2\right)}{\left(\omega_0^2-\omega_c^2\right)^2\left(\omega_0^2-\omega_u^2\right)}. \tag{15}$$

Using the WKB approximation and following the procedure used by Sodha *et al* [2], one can write

$$E_y E_y^* = \frac{E_0^2}{f^2}\exp\left(-\frac{z^2}{r_0^2 f^2}\right) \tag{16}$$

Where $r_o$ is the initial radius and $f$ is known as the beam width parameter governed by the equation

$$\frac{d^2 f}{dx^2} = \frac{1}{k_0^2 r_0^4 f^3} - \frac{\in_2 E_0^2}{\in_{\text{eff}} r_0^2 f^2}. \tag{17}$$

To solve eq. (17) for an initially plane wave front, we assume the following initial condition for $f$ :

$f = 1$ at $x = 0$ and $df/dx = 0$ at $x = 0$,

the latter condition means a plane wave front while the former equates the amplitude distribution at $x = 0$, to the known form. Then the solutions of eq. (17) for $f$ is

$$-\frac{1}{a}\left(bf - af^2 - 1\right)^{1/2} + \frac{b}{2a^{3/2}}\left(\frac{\pi}{2} - \sin^{-1}\frac{2af-b}{b-2}\right) = \frac{x}{R_d} = \xi, \tag{18}$$

where $a = b - 1$, $b = 2R_d^2/R_n^2$, $R_d = k_o r^2$ and $R_n = r_0 (\in_{\text{eff}}/\in_2 E_0^2)^{1/2}$. To show explicitly the variation of beam width parameter $f$ with the distance of propagation, we have plotted $f$ as a function of $z$ in Figure 1 for a typical set of parameters :

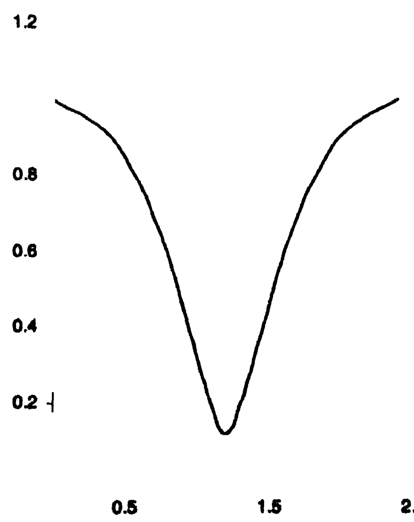


**Figure 1.** Variation of beam width parameter $f$ with distance of propagation $\xi$ for $\omega_p^2/\omega_0^2 = 0.5$, $\omega_c^2/\omega_0^2 = 0.2$, and $(\omega_0^2/c^2)\, r_0^2\, \alpha E_0^2 = 2.6$.

$$\frac{\omega_0^2 r_0^2 \alpha E_0^2}{c^2} = 2.6, \ \frac{\omega c^2}{\omega_0^2} = 0.2$$

and $\frac{\omega_p}{\omega_0^2} = 0.5$, respectively.

One interesting feature of this plot is that $f$ is a periodic function of $x$. The self-focusing of the beam occurs when the second term (due to nonlinear convergence) on the right-hand side exceeds the first term (due to diffraction divergence) on the right-hand side. In a special case when the diffraction divergence term (second in eq. (15) cancels with the focusing term (first in eq. (15)), the beam width, initially plane wavefront propagates as such. For a given power, the beam diameter for uniform wave guide propagation (*i.e.* $f$ is 1 for all values of $x$) is given by the minimum value of $f$, *i.e.*

$$r_{0c} = k_0^{-1}(2\epsilon_{\text{eff}}/\epsilon_2 E_0^2)^{1/2}.$$

The minimum value of $f$, *i.e.*. (19)

$f_{\min} = \epsilon_{\text{eff}}/(2k_0^2 r_0^2 \epsilon_2 E_0^2)$, decreases with increasing $E_0^2$. The variation of $f_{\min}$ with $\omega_c/\omega_0$ is displayed in Figure 2. It may be observed from Figure 2 that $f_{\min}$ decreases with magnetic field.

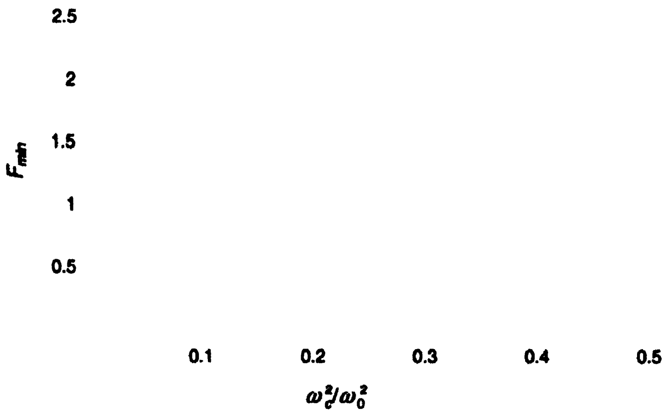


**Figure 2.** Variation of minimum beam width parameter ($F_{\min} = f_{\min}(2\omega_0^2/c^2) r_0^2 \alpha E_0^2$) as function of static magnetic field for $\omega_p^2/\omega_0^2 = 0.5$.

## 4. Filamentatation instability

In this section, we study the growth of a perturbation in the intensity distribution of a uniform laser beam [1–3]. In the presence of the perturbation, the total intensity becomes non-uniform in a plane transverse to the beam propagation and the carriers get redistributed leading to a nonlinear dielectric tensor of the magnetoplasma. In the presence of a perturbation, the total electric vector can be written as

$$E = (E_0 + E_1(x,z))\exp[i(\omega_0 t - k_0 x)], \tag{20}$$

where $E_1(<<E_0)$ refers to the perturbation. The two-dimensional wave equation for the perturbation field may be rewritten as [1]

$$\frac{\partial^2 E_{ly}}{\partial x^2} + \frac{\partial^2 E_{ly}}{\partial z^2} + k_0^2 E_{ly} \cdot \frac{\omega_0^2}{c^2} \epsilon_2 E_{0y}^2 (E_{ly} + E_{ly}^*) = 0 \tag{21}$$

On taking the spatial variations of $E_{ly}$ as exp $[-(k_x x + k_z z)]$ and following Sharma [4] and Sodha *et al* [1]. The growth rate $\gamma (= ik_x)$ of perturbation in the limit $k_x^2 < k_z^2$ can be obtained as

$$\gamma = ik_x$$

$$= k_x / 2k_0 \quad 2k_0^2 \frac{\epsilon_2 E_{0y}}{\epsilon_{\text{eff}}} | - k_z^2 \tag{22}$$

$\gamma$ has the maximum value of

$$\gamma_{\max} = k_0 \epsilon_2 E_{0y}^2 / 2\epsilon_{\text{eff}} \tag{23}$$

for

$$k_x|_{\text{opt}} = k_0(\epsilon_2 E_{0y}^2/\epsilon_{\text{eff}})^{1/2}. \tag{24}$$

The variation of maximum growth rate as a function of magnetic field is shown in Figure 3. The growth rate increases rapidly with $\omega_c/\omega_0$.

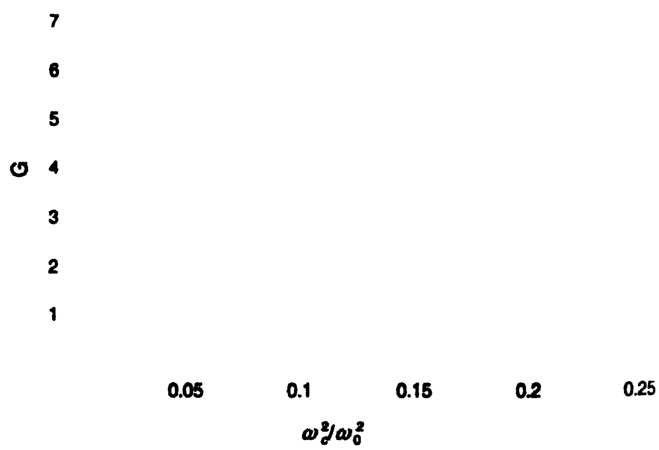


**Figure 3.** Maximum growth rate ($G = \gamma_{\max} 2c/\omega \alpha_0 E_0^2$) of filamentation instability as a function of static magnetic field for $\omega_p^2/\omega_0^2 = 0.5$.

## 5. Discussion

In the presence of high power Gaussian laser beam, propagating along the $x$-axis (static magnetic field being in $z$-direction), the electrons get redistributed in a plane transverse to the direction of pump on account of nonuniform heating.

Under this modified profile of dielectric constant, it is seen that a two-dimensional beam propagates in an oscillatory waveguide. The value of $f_{min}$ decrerses with increasing magnetic field. Thus, the magnetic field and ohmic nonlinearity enhances the rate of self-focusing. It is also seen that laser radiation propagating in an extraordinary mode is unstable to small scale fluctuations and spatial growth rate of filamentation instability increases with an increase in magnetic field.

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